CONVECTIVE DRYING OF DISPERSE MATERIAL IN AN AGITATED LAYER IN CROSS FLOW OF COOLANTS

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A mathematical description is given of the process, and the relationship between the temperature and the moisture content of the material is established; a formula is derived to determine the equipment size for a prespecified amount of moisture to be removed.

A method of calculating the kinetics of the process of convective drying of thin materials in direct flow and countercurrent flow of the material to be dried and the drying agent has been presented [1], the principal feature of the method being the introduction of an external mass-transfer effectiveness factor expressing that fraction of the total amount of heat delivered to the material which is used up in evaporating moisture from the material.

This method is extended, in the present article, to calculations of the kinetics of drying in an agitated layer of granular material. The process equipment is shown in schematic form in Fig. 1.

The case of heat transfer complicated by mass transfer with cross flow of the heat-transfer media (coolants), one of which is agitated in planes perpendicular to its flow, is considered here.

The simplifying assumptions entertained in our treatment reduce to the following:

1. The temperature of the material is a function of the longitudinal coordinate alone, i.e., $t'' = f(x)$, because of intense mixing.
2. Heat transfer between particles of the material and of the gas does not occur in the direction of the x axis.

Stationary heat transfer in cross flow at a 90° angle, with one of the coolants present (the material itself in this case) and evenly distributed throughout the volume of the heat source, is described by the following system of equations [2]:

$$\begin{aligned} \frac{\alpha s_1}{c_1\rho_1 f_1}(t''-t') &= w_{1x}\frac{\partial t'}{\partial x} + w_{1y}\frac{\partial t'}{\partial y}, \\ \frac{\alpha s_2}{c_2\rho_2 f_2}(t'-t'') + \frac{q_2}{\rho_2} &= w_{2x}\frac{\partial t''}{\partial x} + w_{2y}\frac{\partial t''}{\partial y}, \end{aligned} \tag{1}$$

where w_{ix} and w_{iy} are the components of the flow velocity along the coordinate axes.

By hypothesis in this problem, $w_{1x} = w_{2y} = 0$, and the perimeter is related, by definition, to the heat-transfer surface by the equations

$$s_1 dy = dF_y;\quad s_2 dx = dF_x.$$

The heat-transfer surface can be expressed in terms of the mean values of the specific surface area σ_{av} and the volume (bulk) mass of the material ρ''_{av} as follows: on the $0 - x$ interval

$$F_x = \sigma_{av} G''_x = \sigma_{av} G_{2av}\frac{x}{w_2} = \sigma_{av} G_{2av}\frac{bHL\rho''_{av}}{bH\rho''_{av} w_2}\frac{x}{L} = \sigma_{av}\rho''_{a} V_0 \frac{x}{L};$$

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and on the $0-y$ interval

$$F_y = \sigma_{av} G''_y = \sigma_{av} \frac{L}{w_2} G_{2av} \frac{y}{H} = \sigma_{av} G_{2av} \frac{bHL\rho''_{av}}{bH\rho''_{av} w_2} \frac{y}{H} = \sigma_{av}\rho''_{av} V_0 \frac{y}{H}.$$

Then, by converting to dimensionless variables, and recalling, as demonstrated in [3], that $f_2/s_2 = 1/\sigma\rho''$, and also $q_2 = \sigma\rho''\alpha\varepsilon(t'-t'')$ [1], we find that

$$\theta' = 1 - \theta'' - \frac{R_{21}}{1+\varepsilon}\frac{d\theta''}{dv_x}, \tag{2}$$

$$\theta'' = 1 - \theta' - \frac{\partial\theta'}{\partial v_y}. \tag{2a}$$

That system of equations is valid for a drying process during which the external mass-transfer effectiveness factor remains constant. It has been shown [1] that the entire process can be broken up into a series of stages (not more than three or four in practice), in order to meet that condition, in each of which stages the variable ε, as well as the heat-transfer coefficient α, can be treated with reasonable accuracy as constants equal to their mean values in the particular stage.

By averaging Eq. (2) with respect to the coordinate v_y, and taking the first of the above assumptions into account, we get

$$\bar{\theta}' = 1 - \theta'' - \frac{R_{21}}{1+\varepsilon}\frac{d\theta''}{\partial v_x}. \tag{3}$$

By solving Eq. (2a) under the condition $\theta'|_{v_y=0} = 0$, we get

$$\theta' = (1-\theta'')(1-e^{-v_y}), \tag{4}$$

and hence the average gas temperature taken over the height of the bed is determined as

$$\bar{\theta}' = (1-\theta'')\left[1 - \frac{1}{v}(1-e^{-v})\right], \tag{5}$$

where v corresponds to the total heat-transfer surface area on that portion of the equipment.

The temperature of the gas at the exit from the bed is determined from Eq. (4) by replacing $v_y = v$, i.e.,

$$\theta'_{\text{K}} = (1-\theta'')(1-e^{-v}). \tag{6}$$

The heat supplied to the material is used up in heating the material and in evaporating moisture from the material, i.e.,

$$Q = Q_{\text{H}} + Q_{ev} = Q_{\text{H}} - \varepsilon Q; \quad (1+\varepsilon)Q = Q_{\text{H}}. \tag{7}$$

The variable Q is determined from the formula

$$Q = \frac{W_1}{v}\int_0^v (t'_{\text{H}} - t'_{\text{K}})dv_x,$$

which, with Eq. (6) taken into account, becomes

$$Q = \frac{W_1}{v}(1-e^{-v})(t'_{\text{H}} - t''_{\text{H}})\int_0^{v_x}(1-\theta'')\,dv_x. \tag{8}$$

Now since

$$dQ_{\text{H}} = c_2 G_2 dt'' = (c_2^c + c_l \bar{u})\,G_2^c dt'', \tag{9}$$

this expression can be integrated only when the temperature dependence of the moisture content is known.

The amount of moisture to be removed from the material on the interval $0-x$ is

$$G_2^c(\bar{u}_{\text{H}} - \bar{u}) = -\frac{\varepsilon}{r}\alpha\sigma\rho'' b\int_0^x\int_0^H (t'-t'')\,dxdy = -\frac{\varepsilon}{r}\frac{W_1}{v}(t'_{\text{H}} - t''_{\text{H}})\int_0^{v_x}\int_0^v (1-\theta'-\theta'')\,dv_x dv_y. \tag{10}$$

Replacing the expression in parentheses by its value taken from Eq. (2a), and recalling Eq. (6), we get

$$\bar{u} = \bar{u}_{\text{H}} + \varepsilon \frac{c_2^{c} R_{12}^{c}}{rv} (t'_{\text{H}} - t''_{\text{H}})(1 - e^{-v}) \int_0^{v_x} (1 - \theta'')\, dv_x,$$

and hence

$$c_2^{c} + c_l \bar{u} = c_2^{c} R_{12}^{c} (1 + \varepsilon) \left\{ \lambda - \frac{\varkappa}{v} (1 - e^{-v}) \int_0^{v_x} (1 - \theta'')\, dv_x \right\}.$$

Substituting the resulting expression into Eq. (9), we find

$$Q_{\text{H}} = W_1 (t'_{\text{H}} - t''_{\text{H}}) (1 + \varepsilon) \int_0^{\theta''} \left\{ \lambda - \frac{\varkappa}{v} (1 - e^{-v}) \int_0^{v_x} (1 - \theta'')\, dv_x \right\} d\theta''. \tag{11}$$

After substitution of the values of Q and Q_H in Eq. (7), and recalling Eqs. (8) and (11), we obtain the equation

$$(1 - e^{-v}) \int_0^{v_x} (1 - \theta'')\, dv_x = v \int_0^{v_x} \left\{ \lambda - \frac{\varkappa}{v} (1 - e^{-v}) \int_0^{v_x} (1 - \theta'')\, dv_x \right\} \frac{d\theta''}{dv_x} dv_x$$

$$= \theta'' \left\{ v\lambda - \varkappa (1 - e^{-v}) \int_0^{v_x} (1 - \theta'')\, dv_x \right\} + \varkappa (1 - e^{-v}) \int_0^{v_x} \theta'' (1 - \theta'')\, dv_x,$$

in which the right-hand part is transformed by integration by parts.

After reducing similar terms,

$$(1 - e^{-v})(1 + \varkappa\theta'') \int_0^{v_x} (1 - \theta'')\, dv_x = v\lambda\theta'' + \varkappa (1 - e^{-v}) \int_0^{v_x} \theta'' (1 - \theta'')\, dv_x.$$

We then differentiate the resulting expression with respect to v_x:

$$v\lambda - \varkappa (1 - e^{-v}) \int_0^{v_x} (1 - \theta'')\, dv_x = (1 - e^{-v}) \frac{1 - \theta''}{d\theta''/dv_x}.$$

Then, after repeating the differentiation, we end up with the nonlinear equation

$$\frac{d^2\theta''}{dv_x^2} + \left(\frac{1}{1 - \theta''} - \varkappa \right) \left(\frac{d\theta''}{dv_x} \right)^2 = 0. \tag{12}$$

The first boundary condition which must be satisfied by the solution of Eq. (12) is

$$\theta''|_{v_x=} = 0. \tag{13}$$

The second boundary condition can be obtained from Eq. (3), if θ' is replaced in that equation by the value from Eq. (5), viz.,

$$\left. \frac{d\theta''}{dv_x} \right|_{v_x=0} = \frac{1}{v} (1 + \varepsilon) R_{12\text{H}} (1 - e^{-v}) = \frac{1 + \varepsilon}{v} \frac{1 - e^{-v}}{R_{21}^{c} (1 + c_l \bar{u}_{\text{H}}/c_2^{c})} = \frac{1}{\lambda v} (1 - e^{-v}). \tag{14}$$

By introducing the new variable $z = d\theta''/dv_x$, we obtain, instead of Eq. (12), the equation

$$\frac{dz}{d\theta''} + \left(\frac{1}{1 - \theta''} - \varkappa \right) z = 0,$$

the solution of which, with the transformed constraint (14)

$$z\,|_{\theta''=0} = \frac{1}{\lambda v} (1 - e^{-v})$$

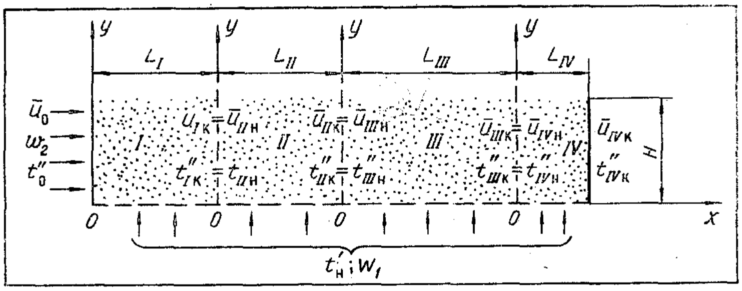

Fig. 1. Diagram of equipment with mixed layer of disperse material. The drying process is broken down into stages designated by Roman numerals. The zero value of the coordinate x corresponds to the beginning of the stage in question.

will be

$$z = \frac{1 - e^{-v}}{\lambda v}(1 - \theta'') \exp(\varkappa\theta'')$$

or, with the replacement of z by its value

$$-\frac{\exp[\varkappa(1-\theta'')]}{\varkappa(1-\theta'')} d[\varkappa(1-\theta'')] = \frac{\exp\varkappa}{\lambda v}(1 - e^{-v})\, dv_x. \tag{15}$$

Upon solving Eq. (15), with the constraint (13), we end up with

$$(1 - e^{-v})\frac{v_x}{v} = \lambda e^{-\varkappa}\{\mathrm{Ei}(\varkappa) - \mathrm{Ei}[\varkappa(1-\theta'')]\}. \tag{16}$$

At the end of the interval $v_x = v$ and $\theta'' = \theta''_K$ of interest here, i.e.,

$$1 - e^{-v} = \lambda e^{-\varkappa}\{\mathrm{Ei}(\varkappa) - \mathrm{Ei}[\varkappa(1-\theta'')]\}. \tag{16a}$$

If we make use of the representation of the integral exponential function in the form of a series, then the expression in the braces in Eq. (16) will acquire the form

$$\mathrm{Ei}(\varkappa) - \mathrm{Ei}[\varkappa(1-\theta'')] = \ln\frac{1}{1-\theta''} + \sum_{n=1}^{\infty}\frac{\varkappa^n[1-(1-\theta'')^n]}{nn!}.$$

It is clear from this last formula that, when $\varkappa = 0$, i.e., when there is no moisture in the material, Eq. (16a) transforms to

$$R_{12}(1 - e^{-v}) = -\ln(1 - \theta''), \tag{16b}$$

i.e., to a formula familiar from the theory of regenerators for cross flow heat exchangers with mixing of one of the heat-transfer media.

Equation (16a) includes two unknowns, θ''_K and v. Accordingly, in order to determine these variables we have to find some other equation relating them, which can be derived from Eq. (10) by replacing the difference $1 - \theta''$ in the integrand by its value from Eq. (15). Then, after integrating and making some transformations, we get

$$\bar{u}_{\text{н}} - \bar{u} = \left(\frac{c_2^{\text{c}}}{c_l} + \bar{u}_{\text{н}}\right)(1 - e^{-\varkappa\theta''}), \tag{17}$$

i.e., the relationship between the volume-average moisture content and the temperature of the material.

The fact that Eq. (17) is identical to an analogous formula cited in [1] for the case of direct flow of heat-transfer media is quite striking. This is evidence that relationships of this type, establishing the exponential behavior of the dependence of the moisture content of a material on the temperature of that material, must be treated as a general regularity in the process of convective drying of thin materials.

From Eqs. (16a) and (17), we can obtain the above unknowns for the temperature of the material at the end of the relevant stage of drying and of the dimensionless heat-transfer surface at that stage.

If we introduce the notation

$$1-\frac{c_2^c/c_l+\bar{u}}{c_2^c/c_l+\bar{u}_н}=\bar{u}^*, \tag{18}$$

then formula (17) can be recast in a different form:

$$\theta_к''=-\frac{1}{\varkappa}\ln(1-\bar{u}_к^*). \tag{19}$$

The specific flow rate of air per kilogram of evaporated moisture can be expressed as follows:

$$l=\frac{G_1}{G_2^c(\bar{u}_н-\bar{u}_к)}=\frac{c_2^c}{c_1}\frac{1}{R_{21}^c(\bar{u}_н-\bar{u}_к)}.$$

On the other hand,

$$l=\frac{1}{d_{2av}-d_1},$$

where d_{2av} and d_1 are respectively the average moisture content of the air exiting from the equipment on that portion of the drying route and the moisture content of the air entering the bed.

From the first two relations, we obtain

$$\lambda=\frac{c_l}{c_1}\frac{d_{2av}-d_1}{\bar{u}_к^*}. \tag{20}$$

The calculations must be carried out in the following sequence.

On the basis of the experimental data, the entire drying process is broken up into stages for each of which the variable $\bar{u}_K^*$ is determined according to Eq. (18), and the value of the external heat-transfer effectiveness coefficient is also determined on the basis of the formula

$$\varepsilon=\frac{[c_l\,(t_н'-t_0'')\,\mathrm{tg}\,\varphi]/r}{1+[c_l\,(t_н'-t_0'')\,\mathrm{tg}\,\varphi]/r}-1, \tag{21}$$

where

$$\mathrm{tg}\,\varphi=d\,[T''/(T_н'-T_0'')]/d\ln[(c_2^c+c_l\,\bar{u}_0)/(c_2^c+c_l\,\bar{u})].$$

Equation (21) is more convenient to use than the analogous formula found in [1], in that it generalizes the results of the experiments carried out at different initial temperature of the drying agent.

The known values of ε, as well as the values of t_H' and t_H'', are used to calculate the value of the parameter $\varkappa$ and, on the basis of Eq. (19), the temperature of the material at the end of the stage in question, θ_K''.

Since the mean temperature of the drying agent at the exit from the bed

$$\theta_{к.av}'=\frac{1}{\theta_к''}\int_0^{\theta_к''}\theta_к'\,d\theta'',$$

then, taking Eq. (6) into account, we have

$$\theta_{к.av}'=(1-e^{-v})\left(1-\frac{1}{2}\theta_к''\right). \tag{22}$$

When the average moisture content of the drying medium exiting from the bed d_{2av} has been specified, the value of λ is found from Eq. (20), which makes it possible to use Eq. (16a) to find the value of the dimensionless coordinate v, which is designated $v_{(16a)}$. On the other hand, the known values of d_{2av}, d_1, and t_H' can be used, on the basis of the J – d diagram, to determine $\theta_{K.av}'$, and it also becomes possible to find the dimensionless coordinate $v_{(22)}$ from Eq. (22), once they are known.

Clearly, equality must prevail in the constraint

$$v_{(16a)}\leqslant v_{(22)}, \tag{23}$$

which can be arrived at through a judicious choice of the average final moisture content of the drying agent upon exit from the bed, d_{2av}. Substitution of that value into Eq. (20) yields the definitive value of λ, and on the basis of the determination of that parameter (see definitions), the specific heat of the drying agent

$$W_1 = \frac{W_2^c}{\lambda}\left(1 + \frac{c_l}{c_2^c}\bar{u}_H\right). \tag{24}$$

The value of W_1 must be verified, subsequently, from the standpoint of bringing about the required hydrodynamic conditions in the bed.

By totalling the values of v for all the intervals in question, we can find the required dimensions of the equipment with ease. It is assumed then that the heat-transfer coefficient is an unknown to be determined from Eq. (16a) by setting up preliminary experiments generalized by a corresponding critical equation.

It is important and mandatory to emphasize the fact that the computational method proposed is of interest solely in the case where the external mass-transfer effectiveness coefficient does not depend appreciably on process condition factors. As demonstrated by Lykov [6, pp. 131-132], that condition is met quite closely in the case of thin materials, and the critical Rb introduced in [4, 5] and related to ε by the straightforward relationship $Rb = 1/(1 + \varepsilon)$, is only a function of the moisture content of the material. This makes it possible to limit the inquiry to setting up only one experiment to find the unknown ε.

NOTATION

α is the heat-transfer coefficient;
ε is the external mass-transfer efficiency factor;
t is the temperature;
w is the flow speed of heat-transfer medium;
ρ is the density;
$f_1 = bL$;
$f_2 = bH$;
L, H see Fig. 1;
b is the width of equipment;
s is the perimeter of heat-transfer surface;
F_x is the heat-transfer surface from entry point of material into interval of interest to the cross section x;
F_y is the same, from entry point of drying medium into bed out to the cross section y;
σ is the surface area presented by a unit mass of the material to be dried;
G'' is the mass of material filling the equipment on the drying interval of interest;
G_2 is the mass flow rate of material;
$V_0 = bHL$;
$\theta' = (t'_H - t')/(t'_H - t''_H)$;
$\theta'' = (t'' - t''_H)/(t'_H - t''_H)$;
$R_{21} = W_2/W_1$;
$W = cG$ is the specific heat of heat-transfer medium;
$R_{21}^c = 1/R_{12}^c = W_2^c/W_1$;
$W_2^c = c_2^c G_2^c$;
$v_x = \alpha\sigma_{av}\rho''_{av}V_0 x/LW_1$;
$v_y = \alpha\sigma_{av}\rho''_{av}V_0 y/HW_1$;
ρ'' is the bulk mass of material;
$v = v_x|_{x=L} = v_y|_{y=H}$;
$\bar{u}$ is the moisture content of material to be dried;
$\bar{u}_0$ is the moisture content at point of entry of material into equipment;
$\bar{u}_H$ is the moisture content at point of entry of material to be dried into interval of interest;
r is the heat of vaporization;
$\varkappa = c_l\varepsilon(t'_H - t''_H)/r(1 + \varepsilon)$;
$\lambda = R_{21}^c(1 + c_l\bar{u}_H/c_2^c)/(1 + \varepsilon)$;
$\bar{u}^*$ see Eq. (18);

c_l	is the specific heat of liquid removed from material being dried;
x, y	are the coordinates.

Subscripts

1, 2	denote the drying medium and material to be dried;
', "	denote the drying medium and material to be dried;
H	denotes the initial value at point of entry to interval of interest;
K	denotes the final value at point of exit from interval of interest;
av	denotes the average taken over the interval of interest.

Superscript

c	denotes the value for absolutely dry material.

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